



Inverse-problem approach for stress-partitioning analysis using stress–strain curve of TRIP steel

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ABSTRACT

An inverse-problem approach is proposed wherein a computational model of the stress–strain (SS) curve of a two-phase composite material is fitted to experimental data on the SS curve of metastable austenitic stainless steel for estimating the SS curves of individual phases of fcc- γ and deformation-induced bcc- α' , as well as the change in the volume fraction of the α' phase ($f^{(\alpha')}$) with increasing strain. The proposed method was applied to the SS curves of two types of TRIP steels: Fe–18Cr–8Ni–0.1C (mass%) alloy (0.1C steel) and Fe–18Cr–8Ni–0.1 N (mass %) alloy (0.1 N steel). The SS curves of individual phases (γ and α') were estimated to reproduce the overall SS curve of TRIP steel reported in a literature. The estimated flow stress of the α' phase for 0.1C steel significantly exceeded that for 0.1 N steel, indicating that carbon addition is more effective than nitrogen addition in strengthening the deformation-induced α' phase. Additionally, the proposed method allowed us to examine the phase stress (stress partitioning between the γ and α' phases) during tensile deformation. Further, the change in $f^{(\alpha')}$ during tensile deformation was estimated, and the results closely matched the experimental data from the literature. The increase rate of $f^{(\alpha')}$ in 0.1C steel is lower than that in 0.1 N steel, which is essential information for optimizing the strength–ductility balance of TRIP steel.

1. Introduction

Applying deformation to a metastable austenitic stainless steel, such as type 304 steel, induces martensitic transformation of the austenite (fcc- γ) phase [1]. The deformation-induced martensitic transformation (DIMIT) contributes to the strengthening of austenitic stainless steel because the strength of the deformation-induced martensite (bcc- α') phase exceeds that of the γ phase. Furthermore, the transformation-induced plasticity (TRIP) effect caused by the DIMIT provides metastable austenitic stainless steel with an excellent strength–ductility balance [2,3]. Several researchers have investigated the effects of interstitial alloying elements on the mechanical properties of austenitic stainless steel [4–9]. The addition of carbon or nitrogen to a stable austenitic stainless steel increases the work-hardening rate in the low-strain region (true strain < 0.25) [4]. In metastable austenitic stainless steel, the addition of carbon or nitrogen, which are γ -stabilizing elements, suppresses DIMIT, increases the work-hardening rate, and strengthens the deformation-induced α' phase [5]. Thus, when designing the mechanical properties of metastable austenitic stainless

steel through microstructure control, it is necessary to quantify the change in the volume fraction of the deformation-induced α' phase ($f^{(\alpha')}$) during deformation, along with the deformation behavior of the individual constituent phases (γ and α').

The change in $f^{(\alpha')}$ during deformation or tensile testing of metastable austenitic stainless steel has been evaluated using X-ray diffraction [2,3,10,11], saturation magnetization [5,12–15], and neutron diffraction measurements [16–18]. The volume fraction of the retained γ phase ($f^{(\gamma)}$) in multiphase TRIP steel has also been evaluated using electron backscatter diffraction measurement [19,20], but this method tends to underestimate $f^{(\gamma)}$ compared to X-ray or neutron diffraction measurements [21]. Additionally, the deformation behavior of the γ and α' phases has been investigated by measuring the hardness of each phase in cold-rolled metastable austenitic stainless steel [5] or measuring the stress partitioning between the two phases during tensile deformation of metastable austenitic stainless steel via *in situ* neutron diffraction experiments [16–18]. Although *in situ* neutron diffraction measurements are effective for evaluating both $f^{(\alpha')}$ and the stress partitioning between the γ and α' phases of metastable austenitic stainless steel, the available

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experimental facilities are limited. Therefore, to systematically investigate the effects of alloying elements on the mechanical properties of metastable austenitic stainless steel, a more practical method that does not require specialized experimental equipment is needed.

Combining experiments and computational modeling of the stress–strain (SS) behavior (macroscopic SS curve) of a multiphase material is a promising approach to understanding the relationship between the mechanical properties and microstructure. TRIP in steel has been simulated using a phase-field model, clarifying the microstructure evolution (formation of a multivariant structure in the α' phase) under external loading [22]. However, owing to the limited size of the computational domain, it is difficult to predict the macroscopic SS curve. The macroscopic deformation behavior of TRIP steel has been predicted via constituent equations that consider the dependence of DIMT on the strain rate, temperature, and applied stress. The constitutive models can reproduce both the SS curve and the change in $f^{(\alpha')}$ with increasing strain, making them useful for numerical analysis of boundary-value problems [23,24]. Meanwhile, the SS curve of TRIP steel has been predicted using Weng's secant method [25], assuming spherical inclusions and focusing on the effects of the mechanical properties of the individual constituent phases (γ and α') on the flow stress of TRIP steel [5,26]. Further, a modified secant method has been proposed for predicting the SS curve of a two-phase composite material containing an inclusion of arbitrary shape, revealing that the mechanical properties of the material depend not only on the volume fraction of the inclusion but also on its shape [27,28].

In the present study, we propose an inverse-problem approach that integrates experimental data on the SS curve of TRIP steel with a computational model (CM) based on the secant method, simultaneously

estimating the flow stresses of individual phases (γ and α') and the change in $f^{(\alpha')}$ with increasing strain. The remainder of this paper is organized as follows. Section 2.1 explains the problem setting. The CM (forward model) that predicts the SS curve of TRIP steel given the SS curves of the individual phases (γ and α') as well as the change in $f^{(\alpha')}$ with increasing strain is presented in Appendixes A.1–A.3. Section 2.2 describes an optimization method for fitting the forward model to experimental data on the SS curve of TRIP steel, and Section 2.3 presents the calculation conditions. In Section 3, the proposed method is applied to experimental data from the literature for validation, and the prediction results for the phase stress (stress partitioning between the γ and α' phases) during tensile deformation are presented. Section 4.1 discusses the initial-value dependence of parameter estimation, and Section 4.2 discusses the effect of the shape or selection of the inclusion phase on the SS curve of TRIP steel. Section 5 presents the main conclusions.

2. Calculation method

2.1. Problem setting

We consider a CM (forward model) that predicts the overall SS curve of the two-phase composite material consisting of the γ and α' phases. It is assumed that the elastic constants of each phase, the SS curves of individual phases, and the change in $f^{(\alpha')}$ with increasing strain are given. Fig. 1(a,b) and (c,d) show the forward and inverse problems, respectively, assumed in the present study. The forward problem involves predicting the overall SS curve of TRIP steel (dashed line in Fig. 1(a)) based on the SS curves of the γ and α' phases (solid lines in Fig. 1(a)) as well as the change in $f^{(\alpha')}$ (solid line in Fig. 1(b)). Meanwhile, the

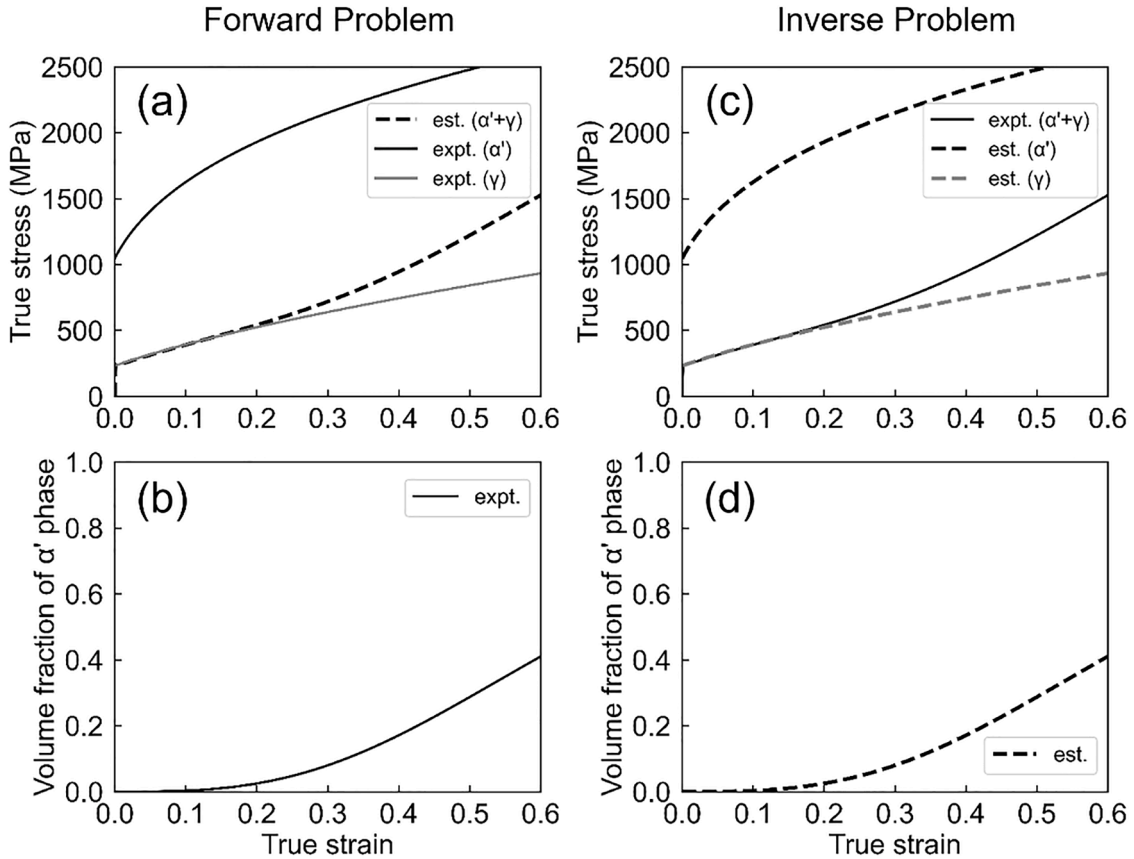


Fig. 1. Forward problem (a,b) and inverse problem (c,d) assumed in the present study. In the forward problem, the SS curve of TRIP steel (dashed line in (a)) is predicted based on the SS curves of the γ and α' phases (solid lines in (a)) as well as the change in the volume fraction of the α' phase ($f^{(\alpha')}$) (solid line in (b)). In the inverse problem, the SS curves of the γ and α' phases (dashed lines in (c)) as well as the change in $f^{(\alpha')}$ (dashed line in (d)) are estimated from the SS curve of TRIP steel (solid line in (c)).

inverse problem involves fitting the forward model to the overall SS curve of TRIP steel (solid line in Fig. 1(c)) for estimating the SS curves of the γ and α' phases (dashed lines in Fig. 1(c)) as well as the change in $f^{(\alpha')}$ (dashed line in Fig. 1(d)).

The CM for predicting the overall SS curve of a two-phase composite material consisting of matrix and inclusion phases is based on the secant method, and its details are presented in Appendixes A.1–A.3 [27–29]. The SS curves of individual phases are approximated by the Swift equation [30]:

$$\sigma^{(r)} = a^{(r)}(b^{(r)} + \varepsilon^{p(r)})^{N^{(r)}} \quad (1)$$

where $\sigma^{(r)}$ and $\varepsilon^{p(r)}$ represent the stress and plastic strain, respectively, and $a^{(r)}$, $b^{(r)}$, and $N^{(r)}$ are material constants, with $r = 0$ for the matrix phase and $r = 1$ for the inclusion phase. Hereinafter, the γ and α' phases are considered the matrix and inclusion phases, respectively. To consider the DMT during tensile testing of TRIP steel, the change in the volume fraction of the inclusion ($f^{(1)}$) is expressed as a function of the overall plastic strain via Matsumura's equation [31]:

$$f^{(1)} = f^{(\alpha')} = 1 - \left(1 + \frac{k}{q}[\varepsilon^{p(C)}]^q\right)^{-1} \quad (2)$$

where k and q are material constants, and $\varepsilon^{p(C)}$ represents the plastic strain of the two-phase composite material. In the forward problem shown in Fig. 1(a,b), the overall SS curve of TRIP steel during tensile deformation is calculated when the parameter $\mathbf{x} = (a^{(0)}, b^{(0)}, N^{(0)}, a^{(1)}, b^{(1)}, N^{(1)}, k, q)^T$ is given.

2.2. Solving inverse problem

The CM for calculating the SS curve of metastable austenitic stainless steel accompanied by DMT is fitted to experimental data on the SS curve of TRIP steel for estimating the SS curves of the γ and α' phases, as well as the change in $f^{(\alpha')}$. Specifically, we consider estimating the parameter $\mathbf{x} = (a^{(0)}, b^{(0)}, N^{(0)}, a^{(1)}, b^{(1)}, N^{(1)}, k, q)^T$, which consists of material parameters of the Swift equation and Matsumura's equation (Eqs. (1) and (2)). A cost function is defined as

$$J(\mathbf{x}) = \frac{1}{M} \sum_{i=1}^M (\bar{\sigma}_{\text{expt},i} - \bar{\sigma}_{\text{calc},i}(\mathbf{x}))^2 \quad (3)$$

where $\bar{\sigma}_{\text{expt},i}$ and $\bar{\sigma}_{\text{calc},i}$ represent the overall stress of TRIP steel obtained via experiment and calculation, respectively, at an identical strain of $\bar{\varepsilon}_i$ (with $i = 1, \dots, M$ denoting measurement points). The optimum $\mathbf{x}$ that minimizes J is determined using SciPy [32] via the Nelder–Mead method [33], without imposing any constraints on the range of values of $\mathbf{x}$.

2.3. Calculation conditions

The proposed method was applied to the experimental data on the SS curves of metastable austenitic stainless steels: Fe–18Cr–8Ni–0.1C (mass %) and Fe–18Cr–8Ni–0.1 N (mass%) alloys (hereinafter referred to as 0.1C steel and 0.1 N steel, respectively) [5]. The experimental data on flow stress were extracted at strain intervals of 0.01 from the SS curves of each steel in the literature [5]. As a result, the number of measurement points was $M = 58$ for 0.1C steel and $M = 51$ for 0.1 N steel. For simplicity, both steels were assumed to be isotropic elastic bodies, and the α' phase was assumed to be spherical. The Young's modulus and Poisson's ratio were set as $E^{(0)} = E^{(1)} = 200$ GPa and $\nu^{(0)} = \nu^{(1)} = 0.3$, respectively [5]. The estimation of $\mathbf{x}$ was started using the 81 ($=3^4$) sets of initial parameter values presented in Table 1. These initial parameter values were determined through trial and error as follows. First, the initial parameter values of the Swift equation for the γ phase were

Table 1

Initial values in parameter estimation for 0.1C steel and 0.1 N steel. A total of 81 ($=3^4$) sets of initial parameter values are used for each steel.

		Swift equation			Matsumura's equation	
		a (MPa)	b	N	k	q
0.1C steel	α'	3000 ± 1000	0.03	0.3 ± 0.1	10 ± 5	3 ± 1
	γ	1200	0.08	0.65		
0.1 N steel	α'	1600 ± 100	0.03	0.4 ± 0.1	200 ± 100	4 ± 0.5
	γ	1200	0.05	0.45		

determined so that the calculated overall SS curve of TRIP steel roughly matched the experimental data in the low-strain region where the value of $f^{(\alpha')}$ was negligibly small. Afterwards, the initial parameter values of the Swift equation for the α' phase and the Matsumura's equation for $f^{(\alpha')}$ were determined so that the calculated overall SS curve of TRIP steel approximately matched the experimental data. As listed in Table 1, three initial values were employed for each of $a^{(1)}$, $N^{(1)}$, k , and q , as they were found to significantly affect the SS curves of both steels in the preliminary analysis. The optimum $\mathbf{x}$ satisfied $J(\mathbf{x}) < 5$ MPa².

3. Results

Table 2 presents the estimated values of $\mathbf{x}$, corresponding to the average values of each parameter when the estimation was successful ($J(\mathbf{x}) < 5$ MPa²) out of 81 parameter estimations. Fig. 2 shows the estimated SS curves of individual phases (γ and α') for (a) 0.1C steel and (c) 0.1 N steel, along with the estimated change in $f^{(\alpha')}$ for (b) 0.1C steel and (d) 0.1 N steel, all calculated using the parameters listed in Table 2. In Fig. 2(a,c), the black and gray dashed lines represent the estimated SS curves of the α' and γ phases, respectively. The calculated overall SS curve of TRIP steel indicated by the black solid line in Fig. 2(a,c) closely reproduces the experimental data (open circles) [5], demonstrating that the CM described in Appendixes A.1–A.3 can be applied to analyze the SS curve of metastable austenitic stainless steel accompanied by DMT. The estimated flow stress of the α' phase differs significantly between 0.1C steel and 0.1 N steel, whereas that of the γ phase does not differ significantly between the two steels. This indicates that the effect of carbon addition on the strengthening of the deformation-induced α' phase is greater than that of nitrogen addition, which is consistent with the findings reported in the literature [5].

The dashed line shown in Fig. 2(b,d) represents the estimated change in $f^{(\alpha')}$, closely reproducing the experimental data (open circles) from the literature [5]. Clearly, the increase rate of $f^{(\alpha')}$ during tensile testing is lower for 0.1C steel than for 0.1 N steel. Several studies have pointed out that suppressing the DMT rate is important to improve the uniform elongation of TRIP steel [18,34]. In fact, due to the gradual increase in the α' phase, 0.1C steel has higher tensile strength and greater uniform elongation than 0.1 N steel, thus exhibiting an excellent strength–ductility balance, as reported in the literature [5]. Note that our inverse-problem approach uses only the experimental data on the overall SS curve of TRIP steel (open circles in Fig. 2(a,c)) and does not use the experimental data on $f^{(\alpha')}$ (open circles in Fig. 2(b,d)). In other words, the proposed inverse-problem approach allows us to obtain information on the change in $f^{(\alpha')}$ from the overall SS curve of TRIP steel.

Table 2

Estimated parameter values of the Swift equation and Matsumura's equation for 0.1C steel and 0.1 N steel.

		Swift equation			Matsumura's equation	
		a (MPa)	b	N	k	q
0.1C steel	α'	2103	0.030	0.13	13.4	2.83
	γ	1382	0.102	0.72		
0.1 N steel	α'	1528	0.029	0.31	238.7	4.28
	γ	1139	0.055	0.41		

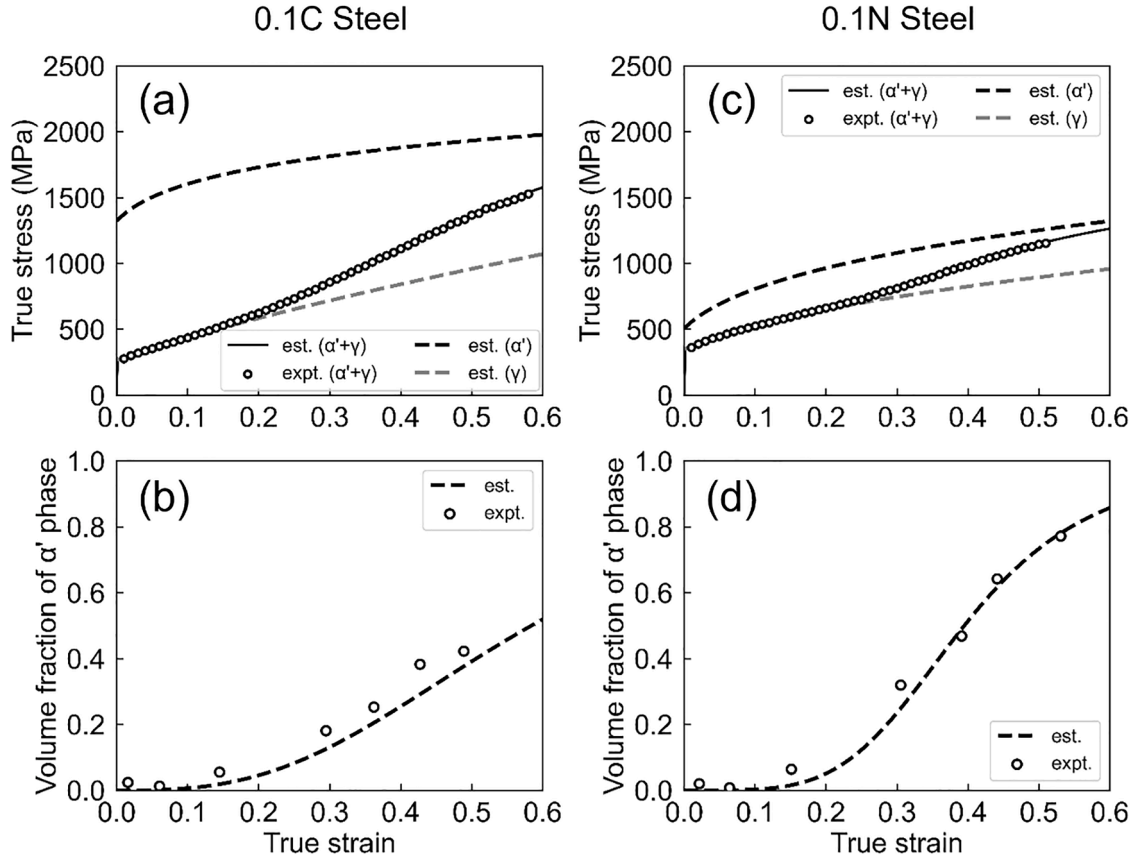


Fig. 2. Estimation results for the flow stresses of individual constituent phases (γ and α') during tensile testing of (a) 0.1C steel and (c) 0.1 N steel, along with the estimated change in the volume fraction of the deformation-induced α' phase ($f^{(\alpha')}$) for (b) 0.1C steel and (d) 0.1 N steel. The solid lines in (a,c) represent the calculated overall SS curves of TRIP steels, the dashed lines in (a–d) represent the estimation results, and the open symbols in (a–d) represent experimental data from the literature [5].

This is a simpler and more practical approach for estimating the change in $f^{(\alpha')}$ during tensile testing than X-ray diffraction measurement [2,3,10,11], saturation magnetization measurement [5,12–15], and neutron diffraction measurement [16–18].

For further validation, the proposed method was also applied to the

experimental data on the SS curves of SUS301L metastable austenitic stainless steel under various temperature and strain rate conditions reported in the literature [3]. The estimation results are summarized in Appendix A.4. The SS curves of the individual α' and γ phases could be estimated to reproduce the overall SS curve reported in the literature

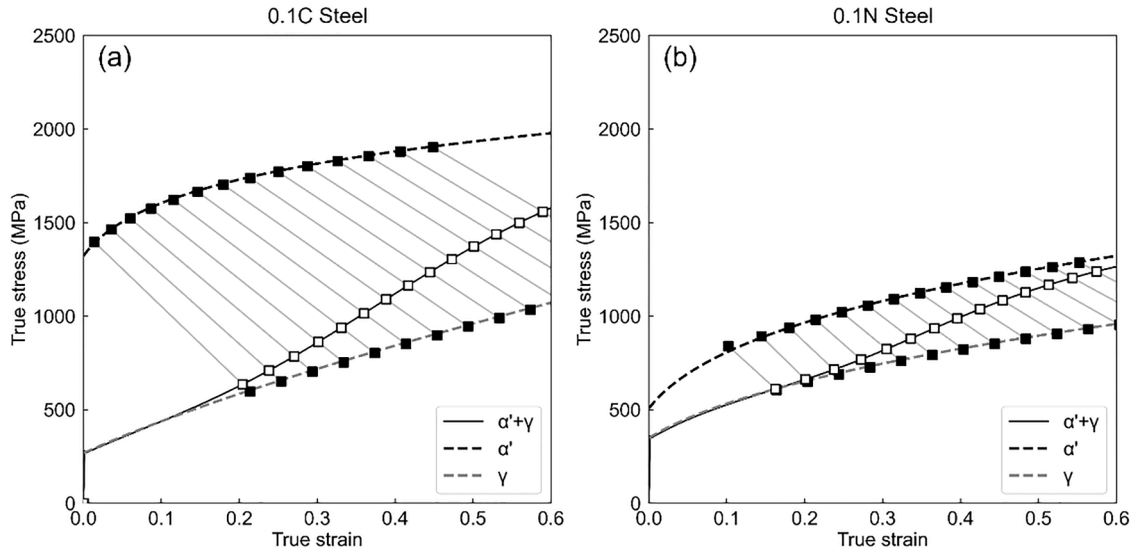


Fig. 3. Calculation results for the phase stress (stress partitioning between the γ and α' phases) during tensile deformation for (a) 0.1C steel and (b) 0.1 N steel. The solid symbols represent the calculated stress of the γ and α' phases. The solid line represents the calculated overall SS curve of TRIP steel, and the dashed lines represent the estimated SS curves of individual phases (γ and α').

[3]. Further, the estimated change in $f^{(\alpha')}$ generally matched experimental data from the literature [3], indicating that the proposed method is widely applicable to metastable austenitic stainless steels. It should be noted that since the experimental data of the overall SS curve of SUS301L steel vary with temperature and strain rate, the estimated SS curves (Swift parameters) of the individual γ and α' phases also changed accordingly (refer to Appendix A.4 for details).

One advantage of the proposed method that fits the CM to the experimental data on the SS curve of TRIP steel is that it allows the calculation of $\sigma^{*(0)}$ and $\sigma^{*(1)}$ during tensile testing (see Eqs. (A.24) and (A.25) in Appendix A.3), enabling analysis of the phase stress (stress partitioning between the γ and α' phases) during tensile deformation. The calculated $\sigma^{*(0)}$ and $\sigma^{*(1)}$ are represented by solid squares in Fig. 3 for (a) 0.1C steel and (b) 0.1 N steel. Further, the calculated overall SS curve of TRIP steel is indicated by a solid line, and the estimated SS curves of the individual phases (γ and α') are indicated by dashed lines. Note that $\sigma^{*(0)}$ and $\sigma^{*(1)}$ do not necessarily lie on the estimated SS curves of the individual γ and α' phases, respectively. The stress partitioning between the γ and α' phases can be visually interpreted by connecting $\sigma^{*(0)}$, $\sigma^{*(1)}$, and the overall stress $\bar{\sigma}$ (open squares) with a straight line, as shown in Fig. 3. The stress of the α' phase significantly exceeds that of the γ phase in 0.1C steel, but such a trend is not observed in 0.1 N steel,

which is consistent with the findings from the aforementioned study [5]. The difference in stress partitioning had a significantly effect on the work-hardening behavior of TRIP steel, as reported in the literature [5].

4. Discussion

4.1. Initial-value dependence of parameter estimation

In the inverse-problem approach, the parameter $\mathbf{x} = (a^{(0)}, b^{(0)}, N^{(0)}, a^{(1)}, b^{(1)}, N^{(1)}, k, q)^T$ was estimated by fitting the CM described in Appendixes A.1–A.3 to experimental data on the SS curve of TRIP steel. The estimation was started using 81 sets of initial parameter values, and it was assumed that the parameter estimation was successful when $J(\mathbf{x}) < 5 \text{ MPa}^2$, based on the reproducibility of the experimental data on the overall SS curve. The success rate of parameter estimation was 42% (=31/81) for 0.1C steel and 90% (=73/81) for 0.1 N steel. The low success rate of parameter estimation for 0.1C steel is attributed to the initial parameter values (initial estimates) listed in Table 1 being less valid than those for 0.1 N steel. Through the inverse-problem approach, it became possible to simultaneously estimate SS curves of individual phases (γ and α'), as well as the change in $f^{(\alpha')}$. The relationship between

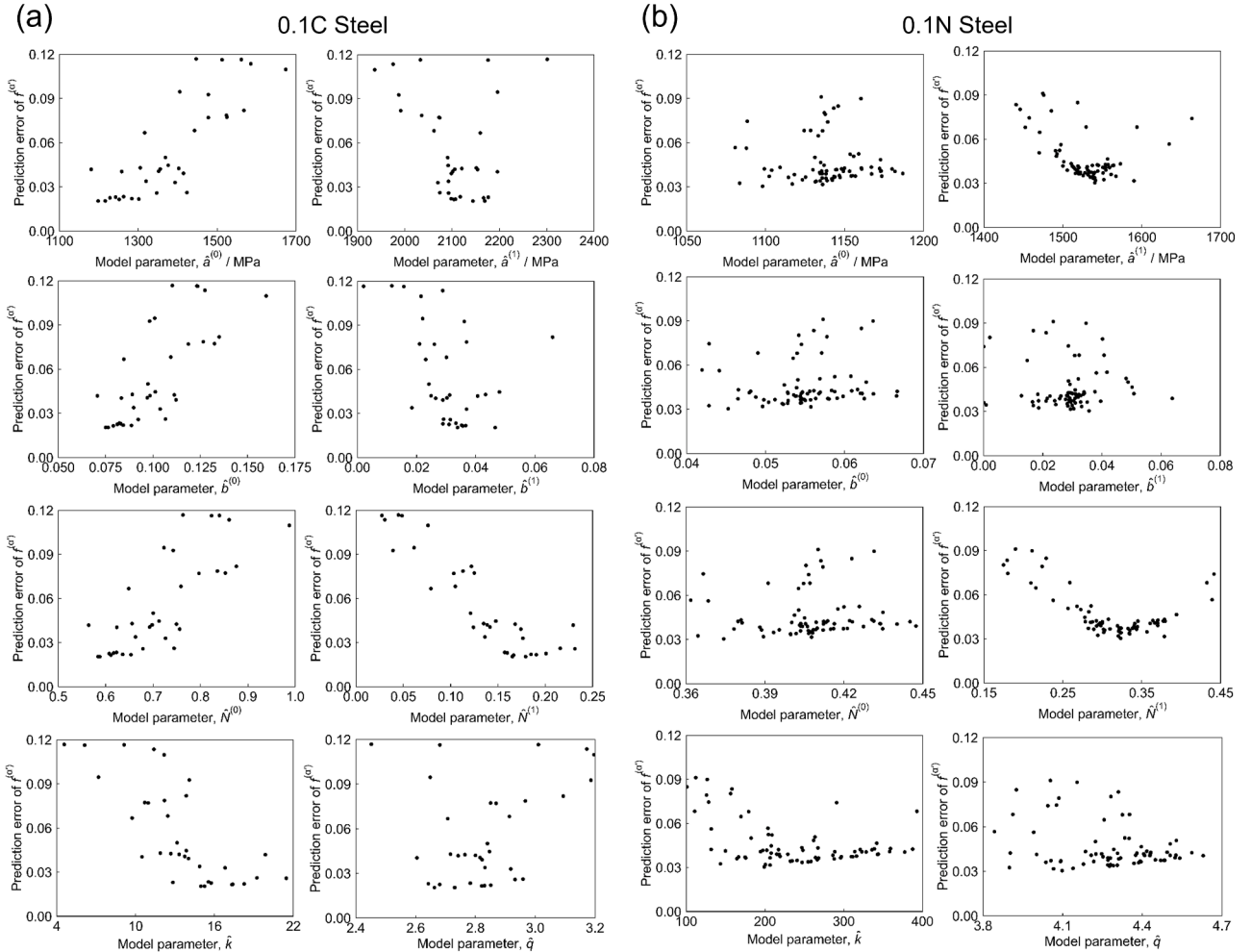


Fig. 4. Plots of the prediction error of the volume fraction of the deformation-induced α' phase against the estimated parameter values: (a) 0.1C steel and (b) 0.1 N steel. Only the results where the parameter estimation was successful, that is, where the overall SS curve of TRIP steel was reproduced, are plotted.

the estimated parameters ($\hat{\mathbf{x}}$) and the prediction error of $f^{(\alpha')}$ was examined. The prediction error of $f^{(\alpha')}$ was defined as

$$J_f(\hat{\mathbf{k}}, \hat{\mathbf{q}}) = \sqrt{\frac{1}{L} \sum_{j=1}^L \left(f_{\text{expt},j}^{(\alpha')} - f_{\text{est},j}^{(\alpha')}(\hat{\mathbf{k}}, \hat{\mathbf{q}}) \right)^2} \quad (4)$$

where $f_{\text{expt},j}^{(\alpha')}$ and $f_{\text{est},j}^{(\alpha')}$ represent the volume fraction of the α' phase obtained via experiment and estimation, respectively, at an identical strain of $\bar{\varepsilon}_j$; $\hat{\mathbf{k}}$ and $\hat{\mathbf{q}}$ are the estimated parameters of Eq. (2). Further, L represents the number of experimental data points, taken from the literature [5]; $L = 7$ for both 0.1C steel and 0.1 N steel. Fig. 4 plots J_f against $\hat{\mathbf{x}}$ for (a) 0.1C steel and (b) 0.1 N steel when the parameter estimation is successful ($J(\mathbf{x}) < 5 \text{ MPa}^2$). Although the values of $\hat{\mathbf{x}}$ vary depending on the set of initial parameter values, the value of J_f is at most 0.12. This indicates that if the overall SS curve of TRIP steel is reproduced, the change in $f^{(\alpha')}$ can be estimated within an error margin of 10%. Meanwhile, because the success rate of parameter estimation depends significantly on the validity of the initial estimates, parameter estimation should be started from multiple sets of initial parameter values. Then, by using the average of the estimated parameters as the final estimate, it is possible to obtain calculation results that reproduce experimental data, as shown in Fig. 2.

4.2. Assumptions regarding the inclusion phase

In the calculation of the overall SS curve of TRIP steel consisting of the γ and α' phases, the α' phase was assumed to be spherical. Considering the detailed process of DIMIT [20,22,35], it may not be reasonable to assume a spherical α' phase in the calculation. Thus, considering spheroidal α' phases with different aspect ratios, we examined the effect of the shape of the α' phase on the overall SS curve of the $\gamma + \alpha'$ two-phase composite material. Fig. 5(a) shows the overall SS curves calculated by fixing the volume fraction of the α' phase at 0.5 and using the material parameters of the Swift equation for 0.1 N steel listed in Table 2. The black, blue, and red plots present the calculation results obtained with the α' phase aspect ratio (R) set to $R = 1$ (sphere), $R = 0.1$ (oblate spheroid), and $R = 10$ (prolate spheroid), respectively. Meanwhile, the orange and green dashed lines represent the SS curves of the individual γ and α' phases, respectively. Fig. 5(b) shows a magnified view of the SS curves of Fig. 5(a) in the low-strain range. The shape of the α' phase

exhibits a slight effect on the overall SS curve in the strain range of 0.001–0.003. However, it has little effect on the overall SS curve in the regions where the strain exceeds 0.003. It was found that the shape of the α' phase affects the overall SS curve when the matrix phase (γ phase) undergoes plastic deformation while the inclusion phase (α' phase) undergoes elastic deformation. Similar results have been obtained in previous studies [27,28].

Another important assumption regarding the inclusion phase in the calculation of the overall SS curve of TRIP steel was that the inclusion phase was the α' phase instead of the γ phase. Using the CM described in Appendixes A.1–A.3, the overall SS curve of a two-phase composite material can be calculated regardless of which phase is considered as the inclusion phase [25,28]. Fig. 6(a) presents the overall SS curves of the $\gamma + \alpha'$ two-phase composite material calculated considering either the α' or γ phase as the inclusion phase. The volume fraction of the α' phase was fixed at 0.5, the inclusion was assumed to be spherical, and the material parameters of the Swift equation for 0.1 N steel listed in Table 2 were used in the calculation. The red plots present the calculation results assuming the α' phase as the inclusion phase, and the blue plots present the calculation results assuming the γ phase as the inclusion phase. Fig. 6(b) shows a magnified view of the SS curves of Fig. 6(a) in the low-strain range. The calculated overall SS curves differ slightly between the two cases only in the strain range of 0.001–0.003, where the γ phase undergoes plastic deformation while the α' phase undergoes elastic deformation.

The calculation results shown in Figs. 5 and 6 indicate that the shape and selection of the inclusion phase only slightly alter the overall SS curve of the $\gamma + \alpha'$ two-phase composite material in the low-strain region. Note that the results shown in Figs. 5 and 6 are for the case where $f^{(\alpha')}$ is fixed at 0.5. The present study focuses on the calculation of the SS curve of TRIP steel, assuming that $f^{(\alpha')}$ increases from zero with the increase in the plastic strain according to Eq. (2). Therefore, in the low-strain region where the α' phase deforms elastically and the γ phase deforms plastically, the value of $f^{(\alpha')}$ is negligibly small. This ensures the effectiveness of the proposed method, which estimates the SS curves of the γ and α' phases as well as the change in $f^{(\alpha')}$ from the overall SS curve of TRIP steel, regardless of the uncertainties in the assumptions regarding the shape or selection of the inclusion phase. On the other hand, the calculation results shown in Fig. 5 also indicate that it is difficult to discuss the effect of the shape of the α' phase on the estimation results in the proposed method.

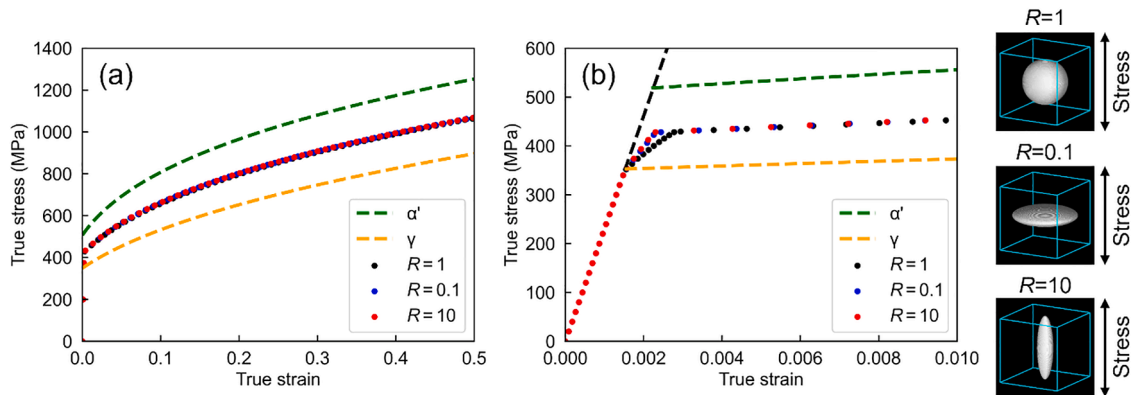


Fig. 5. (a) SS curves of a two-phase composite material consisting of the γ and α' phases, calculated assuming that the α' phase has a spheroidal shape with an aspect ratio of $R = 1$, $R = 0.1$, or $R = 10$. The volume fraction of the α' phase is fixed at 0.5, and the SS curves of individual γ and α' phases (dashed lines) for 0.1 N steel (material parameters of the Swift equation listed in Table 2) are used. (b) Magnified view of the SS curves of (a) in the low-strain range.

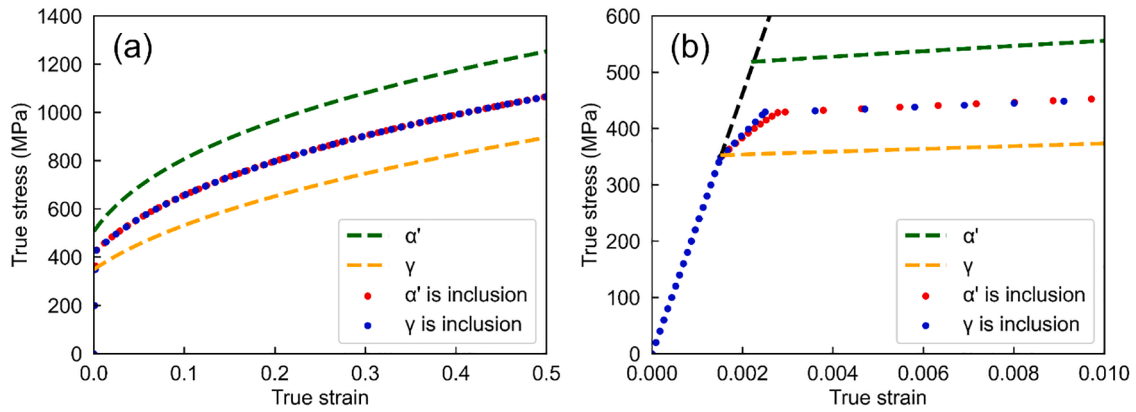


Fig. 6. (a) SS curves of a two-phase composite material consisting of the γ and α' phases, calculated assuming either the α' or γ phase as the inclusion phase. The volume fraction of the α' phase is fixed at 0.5, the inclusion phase is assumed to be spherical, and the SS curves of individual γ and α' phases (dashed lines) for 0.1 N steel (material parameters of the Swift equation listed in Table 2) are used. (b) Magnified view of the SS curves of (a) in the low-strain range.

5. Conclusions

A computational model of the SS curve of a two-phase composite material was fitted to experimental data on the SS curves of Fe–18Cr–8Ni–0.1C (0.1C steel) and Fe–18Cr–8Ni–0.1 N (0.1 N steel) alloys [5]. Thus, we estimated the SS curves of the γ and α' phases and the change in the volume fraction of the deformation-induced α' phase ($f^{(\alpha')}$). The following conclusions are drawn.

- 1 The estimated SS curves of individual constituent phases (γ and α') closely reproduced the experimental data on the overall SS curves for 0.1C steel and 0.1 N steel. Further, the estimated change in $f^{(\alpha')}$ during tensile deformation closely reproduced experimental data from the literature [5]. The success rate of estimating model parameters depended significantly on the validity of the initial estimates. Therefore, starting the estimation from multiple sets of initial parameter values and using the average of the estimated parameter values as the final estimate is effective for obtaining calculation results that reproduce the experimental data.
- 2 The estimated flow stress of the α' phase differs significantly between 0.1C steel and 0.1 N steel, indicating that the effect of carbon addition on the strengthening of the deformation-induced α' phase is greater than that of nitrogen addition. Further, the increase rate of $f^{(\alpha')}$ for 0.1C steel was lower than that for 0.1 N steel, which is essential information for optimizing the strength–ductility balance of TRIP steel. These results were consistent with the findings reported in the literature [5].
- 3 The inverse-problem approach also allowed analysis of the phase stress (stress partitioning between the γ and α' phases) during tensile deformation. The stress of the α' phase was significantly higher than

that of the γ phase in 0.1C steel, but such a trend was not confirmed in 0.1 N steel.

The proposed inverse-problem approach provides a practical way to extract information on the DIMT rate and the stress partitioning between the γ and α' phases from experimental data on the SS curve of metastable austenitic stainless steel. Therefore, this method provides a deeper understanding of the effects of alloying elements on the mechanical properties of TRIP steel, paving the way for optimizing the strength–ductility balance of TRIP steel.

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CRediT authorship contribution statement

Reo Kawamoto: Writing – original draft, Visualization, Software, Methodology. **Yuhki Tsukada:** Writing – review & editing, Supervision, Methodology, Funding acquisition, Conceptualization. **Toshiyuki Koyama:** Writing – review & editing. **Dasom Kim:** Writing – review & editing. **Naoki Takata:** Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix

A.1 Basic relations in the elastic regime

Let us consider a two-phase composite material consisting of matrix and inclusion phases. First, consider the case where both the matrix and the inclusion undergo elastic deformation. Let $\mathbf{C}^{(0)}$, $\mathbf{C}^{(1)}$, and $\mathbf{C}^{(C)}$ be the elastic-modulus tensor, $\boldsymbol{\sigma}^{(0)}$ and $\boldsymbol{\sigma}^{(1)}$ be the stresses, and $\boldsymbol{\varepsilon}^{(0)}$ and $\boldsymbol{\varepsilon}^{(1)}$ be the strains, where 0, 1, and C inside the superscript parentheses indicate the matrix, inclusion, and two-phase composite material, respectively. Further, let $\bar{\boldsymbol{\sigma}}$ and $\bar{\boldsymbol{\varepsilon}}$ be the overall stress and strain, respectively, of the two-phase composite material. The stress–strain (SS) relation (Hooke's law) is schematically shown

in Fig. A.1 [25], where $\sigma^{\text{pt}} \equiv \sigma^{(1)} - \sigma^{(0)}$, $\epsilon^{\text{pt}} \equiv \epsilon^{(1)} - \epsilon^{(0)}$, $\tilde{\sigma} \equiv \sigma^{(0)} - \bar{\sigma}$, $\epsilon^0 \equiv [\mathbf{C}^{(0)}]^{-1} \bar{\sigma}$, and $\tilde{\epsilon} \equiv \epsilon^{(0)} - \epsilon^0$ [36,37]. The variables $\sigma^{(0)}$ and $\sigma^{(1)}$ can be expressed as follows:

$$\sigma^{(0)} = \bar{\sigma} + \tilde{\sigma} = \mathbf{C}^{(0)} \epsilon^{(0)} = \mathbf{C}^{(0)} (\epsilon^0 + \tilde{\epsilon}) \quad (\text{A.1})$$

$$\sigma^{(1)} = \bar{\sigma} + \tilde{\sigma} + \sigma^{\text{pt}} = \mathbf{C}^{(1)} \epsilon^{(1)} = \mathbf{C}^{(1)} (\epsilon^0 + \tilde{\epsilon} + \epsilon^{\text{pt}}) \quad (\text{A.2})$$

Further, based on Eshelby's equivalence principle, $\sigma^{(1)}$ can be expressed in terms of $\mathbf{C}^{(0)}$ as

$$\sigma^{(1)} = \mathbf{C}^{(1)} (\epsilon^0 + \tilde{\epsilon} + \epsilon^{\text{pt}}) = \mathbf{C}^{(0)} (\epsilon^0 + \tilde{\epsilon} + \epsilon^{\text{pt}} - \epsilon^*) \quad (\text{A.3})$$

where ϵ^* represents Eshelby's equivalent transformation strain. Then, substituting Eqs. (A.1) and (A.2) into Eq. (A.3) yields

$$\sigma^{\text{pt}} = \mathbf{C}^{(0)} (\epsilon^{\text{pt}} - \epsilon^*) \quad (\text{A.4})$$

The variable ϵ^{pt} is related to ϵ^* through Eshelby's tensor $\mathbf{S}^{(0)}$:

$$\epsilon^{\text{pt}} = \mathbf{S}^{(0)} \epsilon^* \quad (\text{A.5})$$

As $\bar{\sigma}$ is equal to the weighted mean of $\sigma^{(0)}$ and $\sigma^{(1)}$, i.e., $\bar{\sigma} = f^{(0)} \sigma^{(0)} + f^{(1)} \sigma^{(1)}$, where $f^{(0)}$ and $f^{(1)}$ represent the volume fractions of the matrix and inclusion, respectively, we obtain

$$\tilde{\sigma} = -f^{(1)} \mathbf{C}^{(0)} (\mathbf{S}^{(0)} - \mathbf{I}) \epsilon^* \quad (\text{A.6})$$

$$\tilde{\epsilon} = -f^{(1)} (\mathbf{S}^{(0)} - \mathbf{I}) \epsilon^* \quad (\text{A.7})$$

Meanwhile, $\bar{\epsilon}$ is determined from the weighted mean of $\epsilon^{(0)}$ and $\epsilon^{(1)}$:

$$\bar{\epsilon} = f^{(0)} \epsilon^{(0)} + f^{(1)} \epsilon^{(1)} = f^{(0)} (\epsilon^0 + \tilde{\epsilon}) + f^{(1)} (\epsilon^0 + \tilde{\epsilon} + \epsilon^{\text{pt}}) = \epsilon^0 + f^{(1)} \epsilon^* \quad (\text{A.8})$$

Substituting Eqs. (A.5) and (A.7) into Eq. (A.3) yields

$$\epsilon^* = \mathbf{A} \epsilon^0 \quad (\text{A.9})$$

$$\mathbf{A} \equiv -\mathbf{P}^{-1} (\mathbf{C}^{(1)} - \mathbf{C}^{(0)})$$

$$\mathbf{P} \equiv (\mathbf{C}^{(1)} - \mathbf{C}^{(0)}) (f^{(1)} \mathbf{I} + f_0 \mathbf{S}^{(0)}) + \mathbf{C}^{(0)}$$

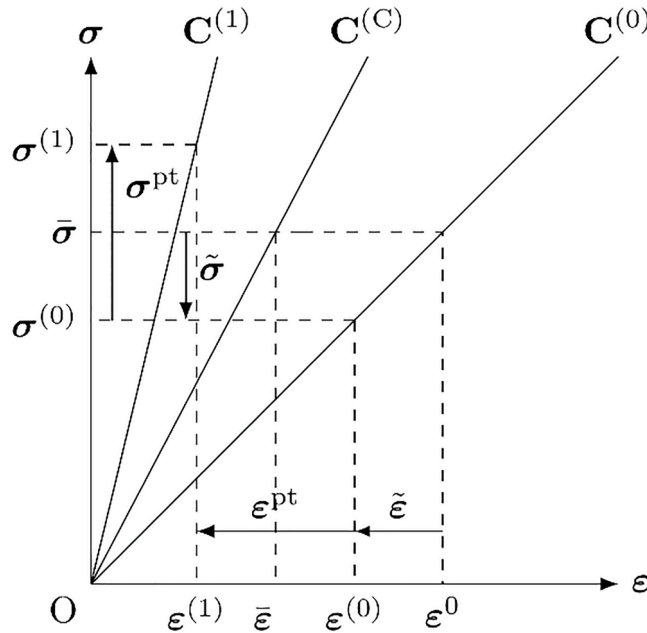


Fig. A.1. Schematic of the SS relation of a two-phase composite material in the elastic regime [25], where $\bar{\sigma}$ and $\bar{\epsilon}$ represent the overall stress and strain of the two-phase composite material, respectively, and 0, 1, and C inside the superscript parentheses indicate the matrix, inclusion, and two-phase composite material, respectively.

A.2 Basic relations in the plastic regime

Next, consider the case where the inclusion undergoes plastic deformation, while the matrix undergoes elastic deformation. Let $\epsilon^{p(1)}$ be the plastic strain of the inclusion. The basic relations can be derived from those in Section A.1. Eq. (A.3) is rewritten as

$$\sigma^{(1)} = \mathbf{C}^{(1)}(\epsilon^0 + \tilde{\epsilon} + \epsilon^{pt} - \epsilon^{p(1)}) = \mathbf{C}^{(0)}(\epsilon^0 + \tilde{\epsilon} + \epsilon^{pt} - \epsilon^{p(1)} - \epsilon^*) \quad (\text{A.10})$$

By replacing ϵ^* with $\epsilon^{p(1)} + \epsilon^*$, Eqs. (A.5), (A.7), and (A.8) are rewritten as follows:

$$\epsilon^{pt} = \mathbf{S}^{(0)}(\epsilon^{p(1)} + \epsilon^*) \quad (\text{A.11})$$

$$\tilde{\epsilon} = -f^{(1)}(\mathbf{S}^{(0)} - \mathbf{I})(\epsilon^{p(1)} + \epsilon^*) \quad (\text{A.12})$$

$$\bar{\epsilon} = \epsilon^0 + f^{(1)}(\epsilon^{p(1)} + \epsilon^*) \quad (\text{A.13})$$

Substituting Eqs. (A.11) and (A.12) into Eq. (A.10) yields

$$\epsilon^{p(1)} + \epsilon^* = \mathbf{A}\epsilon^0 + \mathbf{P}^{-1}\mathbf{C}^{(1)}\epsilon^{p(1)} \quad (\text{A.14})$$

Then, $\sigma^{(0)}$ and $\sigma^{(1)}$ are calculated using Eqs. (A.1) and (A.10), respectively:

$$\sigma^{(0)} = \left\{ \mathbf{I} + f^{(1)}\mathbf{C}^{(0)}(\mathbf{I} - \mathbf{S}^{(0)})\mathbf{A}[\mathbf{C}^{(0)}]^{-1} \right\} \bar{\sigma} + f^{(1)}\mathbf{C}^{(0)}(\mathbf{I} - \mathbf{S}^{(0)})\mathbf{P}^{-1}\mathbf{C}^{(1)}\epsilon^{p(1)} \quad (\text{A.15})$$

$$\sigma^{(1)} = \mathbf{C}^{(1)}\left\{ \mathbf{I} + (f^{(1)}\mathbf{I} + f^{(0)}\mathbf{S}^{(0)})\mathbf{A} \right\} [\mathbf{C}^{(0)}]^{-1} \bar{\sigma} - \mathbf{C}^{(1)}\left\{ \mathbf{I} - (f^{(1)}\mathbf{I} + f^{(0)}\mathbf{S}^{(0)})\mathbf{P}^{-1}\mathbf{C}^{(1)} \right\} \epsilon^{p(1)} \quad (\text{A.16})$$

Further, $\epsilon^{p(1)}$ is calculated using the Lévy–Mises equation [38,39]:

$$\epsilon^{p(1)} = \mathbf{D}^{(1)}\sigma^{(1)} \quad (\text{A.17})$$

$$[D_{ij}^{(1)}] = \frac{\epsilon^{p*(1)}}{\sigma^{*(1)}} \begin{pmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} & 0 & 0 & 0 \\ -\frac{1}{2} & 1 & -\frac{1}{2} & 0 & 0 & 0 \\ -\frac{1}{2} & -\frac{1}{2} & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 \end{pmatrix}$$

where $\sigma^{*(1)}$ and $\epsilon^{p*(1)}$ represent the equivalent stress and equivalent plastic strain, respectively, of the inclusion. Substituting Eq. (A.17) into Eqs. (A.15) and (A.16) yields

$$\sigma^{(1)} = \mathbf{X}^{(1)}\bar{\sigma} \quad (\text{A.18})$$

$$\mathbf{X}^{(1)} \equiv [\mathbf{I} - \mathbf{B}^{(1)}\mathbf{D}^{(1)}]^{-1}\mathbf{A}^{(1)}$$

$$\mathbf{A}^{(1)} \equiv \mathbf{C}^{(1)}\left\{ \mathbf{I} + (f^{(1)}\mathbf{I} + f^{(0)}\mathbf{S}^{(0)})\mathbf{A} \right\} [\mathbf{C}^{(0)}]^{-1}$$

$$\mathbf{B}^{(1)} \equiv -\mathbf{C}^{(1)}\left\{ \mathbf{I} - (f^{(1)}\mathbf{I} + f^{(0)}\mathbf{S}^{(0)})\mathbf{P}^{-1}\mathbf{C}^{(1)} \right\}$$

$$\sigma^{(0)} = \mathbf{X}^{(0)}\bar{\sigma} \quad (\text{A.19})$$

$$\mathbf{X}^{(0)} \equiv \mathbf{A}^{(0)} + \mathbf{B}^{(0)}\mathbf{D}^{(1)}\mathbf{X}^{(1)}$$

$$\mathbf{A}^{(0)} \equiv \mathbf{I} + f^{(1)}\mathbf{C}^{(0)}(\mathbf{I} - \mathbf{S}^{(0)})\mathbf{A}[\mathbf{C}^{(0)}]^{-1}$$

$$\mathbf{B}^{(0)} \equiv f^{(1)}\mathbf{C}^{(0)}(\mathbf{I} - \mathbf{S}^{(0)})\mathbf{P}^{-1}\mathbf{C}^{(1)}$$

Then $\bar{\epsilon}$ is calculated using Eq. (A.13):

$$\bar{\epsilon} = \mathbf{F}\bar{\sigma} \quad (\text{A.20})$$

$$\mathbf{F} \equiv (\mathbf{I} + f^{(1)}\mathbf{A})[\mathbf{C}^{(0)}]^{-1} + f^{(1)}\mathbf{P}^{-1}\mathbf{C}^{(1)}\mathbf{D}^{(1)}\mathbf{X}^{(1)}$$

When the matrix undergoes plastic deformation, $\mathbf{C}^{(0)}$ in the above equations is replaced by the secant elastic modulus $\mathbf{C}^{S(0)}$ [25]. For an isotropic matrix, the secant Young's modulus $E^{S(0)}$ and the secant Poisson's ratio $\nu^{S(0)}$ are expressed as follows:

$$E^{S(0)} = \left(\frac{1}{E^{(0)}} + \frac{\varepsilon^{p*(0)}}{\sigma^{*(0)}} \right)^{-1} \quad (\text{A.21})$$

$$\nu^{S(0)} = \frac{1}{2} - \left(\frac{1}{2} - \nu^{(0)} \right) \frac{E^{S(0)}}{E^{(0)}} \quad (\text{A.22})$$

where $E^{(0)}$, $\nu^{(0)}$, $\sigma^{*(0)}$, and $\varepsilon^{p*(0)}$ represent the Young's modulus, Poisson's ratio, equivalent stress, and equivalent plastic strain, respectively, of the matrix.

The plastic strain of the two-phase composite material is determined by subtracting the elastic strain from the total strain ($\bar{\varepsilon}$):

$$\varepsilon^{p(C)} = \mathbf{F}^p \bar{\varepsilon} \quad (\text{A.23})$$

$$\mathbf{F}^p \equiv (\mathbf{I} + f^{(1)}\mathbf{A})[\mathbf{C}^{S(0)}]^{-1} + f^{(1)}\mathbf{P}^{-1}\mathbf{C}^{(1)}\mathbf{D}^{(1)}\mathbf{X}^{(1)} - (\mathbf{I} + f^{(1)}\mathbf{A}^e)[\mathbf{C}^{(0)}]^{-1}$$

where the superscript e indicates the quantity in the elastic regime.

A.3 Calculation of overall stress-strain curve

The SS curves of individual phases are approximated by the Swift equation (Eq. (1)). The yield criteria for the matrix and inclusion are given by $\sigma^{*(0)} = Y^{(0)}$ and $\sigma^{*(1)} = Y^{(1)}$, respectively, where the yield stress $Y^{(r)}$ is calculated by substituting $\varepsilon^{p(r)} = 0$ into Eq. (1). Hereinafter, the γ and α' phases are considered the matrix and inclusion phases, respectively. To consider the DIMT during tensile testing of TRIP steel, the change in the volume fraction of the α' phase ($f^{(1)}$) is expressed via Matsumura's equation (Eq. (2)). Fig. A.2 presents a flowchart for calculating the SS curve of metastable austenitic stainless steel accompanied by DIMT. The calculation proceeds in the order of Stages I, II, and III. In Stage I, both the matrix (γ) and inclusion (α') undergo elastic deformation. In Stage II, the matrix (γ) undergoes plastic deformation while the inclusion (α') undergoes elastic deformation. In Stage III, both the matrix (γ) and inclusion (α') undergo plastic deformation. Under the uniaxial tensile stress $\bar{\sigma}$, $\sigma^{*(0)}$ and $\sigma^{*(1)}$ are expressed as follows:

$$\sigma^{*(0)} = \left[\frac{1}{2} \left\{ (X_{11}^{(0)} - X_{21}^{(0)})^2 + (X_{11}^{(0)} - X_{31}^{(0)})^2 + (X_{21}^{(0)} - X_{31}^{(0)})^2 \right\} + 3 \left\{ (X_{41}^{(0)})^2 + (X_{51}^{(0)})^2 + (X_{61}^{(0)})^2 \right\} \right]^{\frac{1}{2}} \bar{\sigma} \quad (\text{A.24})$$

$$\sigma^{*(1)} = \left[\frac{1}{2} \left\{ (X_{11}^{(1)} - X_{21}^{(1)})^2 + (X_{11}^{(1)} - X_{31}^{(1)})^2 + (X_{21}^{(1)} - X_{31}^{(1)})^2 \right\} + 3 \left\{ (X_{41}^{(1)})^2 + (X_{51}^{(1)})^2 + (X_{61}^{(1)})^2 \right\} \right]^{\frac{1}{2}} \bar{\sigma} \quad (\text{A.25})$$

Further, the equivalent strain and equivalent plastic strain are expressed as follows:

$$\bar{\varepsilon} = \left[\frac{2}{9} \left\{ (F_{11} - F_{21})^2 + (F_{11} - F_{31})^2 + (F_{21} - F_{31})^2 \right\} + \frac{1}{3} \left\{ (F_{41})^2 + (F_{51})^2 + (F_{61})^2 \right\} \right]^{\frac{1}{2}} \bar{\sigma} \quad (\text{A.26})$$

$$\varepsilon^{p*(C)} = \left[\frac{2}{9} \left\{ (F_{11}^p - F_{21}^p)^2 + (F_{11}^p - F_{31}^p)^2 + (F_{21}^p - F_{31}^p)^2 \right\} + \frac{1}{3} \left\{ (F_{41}^p)^2 + (F_{51}^p)^2 + (F_{61}^p)^2 \right\} \right]^{\frac{1}{2}} \bar{\sigma} \quad (\text{A.27})$$

In Stage III, $\varepsilon^{p*(1)}$ is determined to satisfy the following equation:

$$\frac{\sigma^{*(1)}}{\sigma^{*(0)}} = \frac{\left[\frac{1}{2} \left\{ (X_{11}^{(1)} - X_{21}^{(1)})^2 + (X_{11}^{(1)} - X_{31}^{(1)})^2 + (X_{21}^{(1)} - X_{31}^{(1)})^2 \right\} + 3 \left\{ (X_{41}^{(1)})^2 + (X_{51}^{(1)})^2 + (X_{61}^{(1)})^2 \right\} \right]^{\frac{1}{2}}}{\left[\frac{1}{2} \left\{ (X_{11}^{(0)} - X_{21}^{(0)})^2 + (X_{11}^{(0)} - X_{31}^{(0)})^2 + (X_{21}^{(0)} - X_{31}^{(0)})^2 \right\} + 3 \left\{ (X_{41}^{(0)})^2 + (X_{51}^{(0)})^2 + (X_{61}^{(0)})^2 \right\} \right]^{\frac{1}{2}}} \quad (\text{A.28})$$

which is derived by dividing Eq. (A.25) by Eq. (A.24).

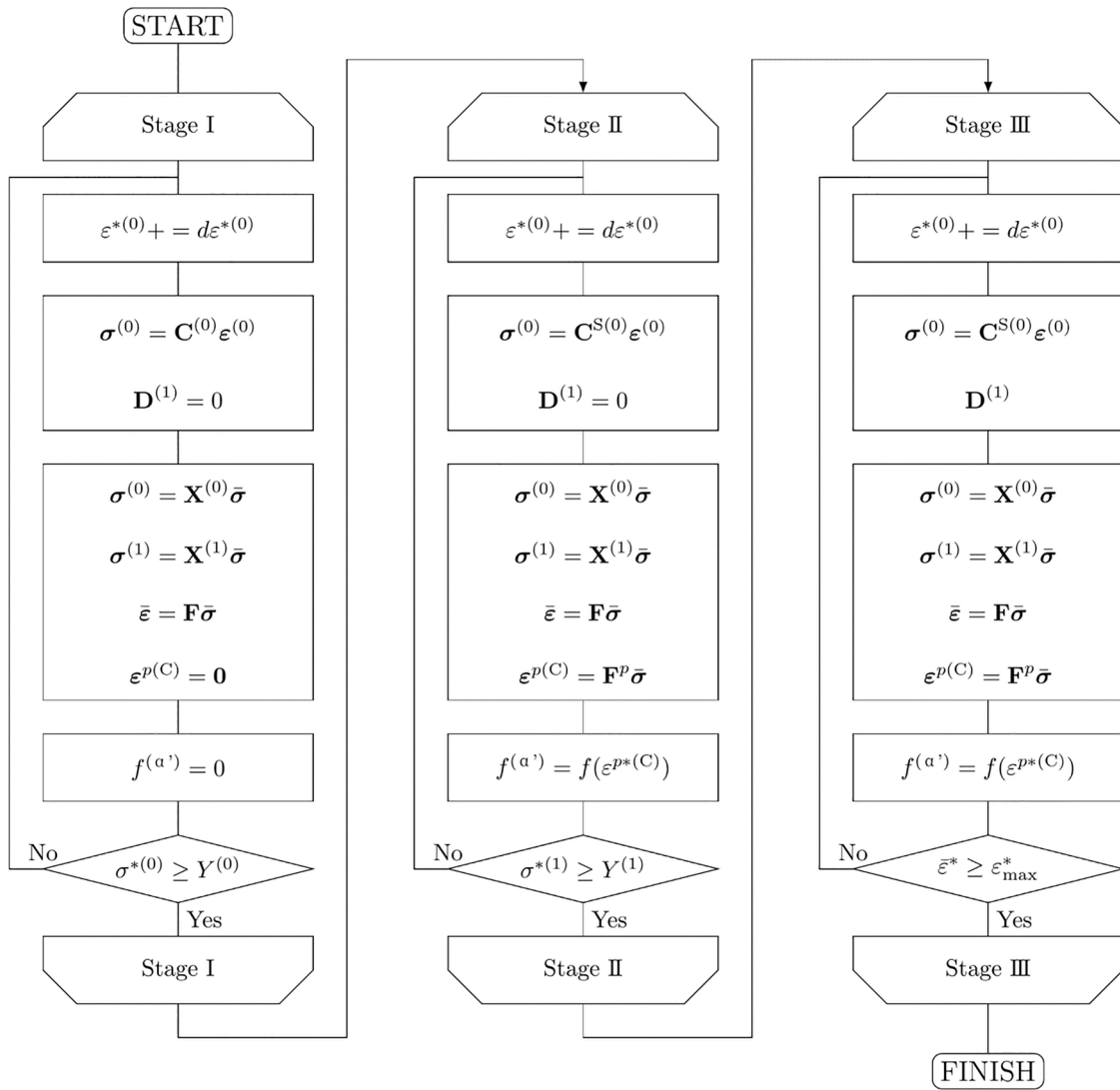


Fig. A.2. Flowchart for calculating the SS curve of metastable austenitic stainless steel accompanied by DMT. Stage I: both the matrix (γ) and inclusion (α') undergo elastic deformation; Stage II: the matrix (γ) undergoes plastic deformation while the inclusion (α') undergoes elastic deformation; Stage III: both the matrix (γ) and inclusion (α') undergo plastic deformation.

A.4 Applicability of the proposed method to SUS301L steel

The proposed method was applied to the experimental data on the stress–strain (SS) curves of SUS301L metastable austenitic stainless steels under various temperature and strain rate conditions reported in the literature [3]. The experimental data on flow stress were extracted at strain intervals of 0.01 from the SS curves in the literature [3]. The estimation of $\mathbf{x} = (a^{(0)}, b^{(0)}, N^{(0)}, a^{(1)}, b^{(1)}, N^{(1)}, k, q)$ was started using the 81 ($=3^4$) sets of initial parameter values presented in Table A.1. Table A.2 presents the estimated values of $\mathbf{x}$, corresponding to the average values of each parameter when the estimation was successful out of 81 parameter estimations.

Table A.1

Initial values in parameter estimation for SUS301L steel. A total of 81 ($=3^4$) sets of initial parameter values are used for each temperature and strain rate condition of tensile tests [3].

Temperature (K)	Strain rate (s^{-1})		Swift equation			Matsumura's equation	
			a (MPa)	b	N	k	q
296	3.3×10^{-4}	α'	2000 ± 500	0.03	0.4 ± 0.1	200 ± 100	4 ± 0.5
		γ	1400	0.05	0.5		
243	3.3×10^{-4}	α'	3000 ± 500	0.03	0.5 ± 0.1	500 ± 100	3 ± 1
		γ	800	0.03	0.18		
323	3.3×10^{-4}	α'	2000 ± 500	0.03	0.4 ± 0.1	100 ± 50	3 ± 1
		γ	1400	0.05	0.5		
296	3.3×10^{-6}	α'	2000 ± 500	0.03	0.4 ± 0.1	200 ± 100	4 ± 0.5
		γ	1400	0.05	0.5		
296	10^{-2}	α'	2000 ± 500	0.03	0.4 ± 0.1	100 ± 50	3 ± 1
		γ	1400	0.05	0.5		

Table A.2

Estimated parameter values of the Swift equation and Matsumura's equation for SUS301L steel.

Temperature (K)	Strain rate (s^{-1})		Swift equation			Matsumura's equation	
			a (MPa)	b	N	k	q
296	3.3×10^{-4}	α'	2038	0.033	0.30	208.0	4.53
		γ	1301	0.041	0.42		
243	3.3×10^{-4}	α'	3437	0.026	0.46	732.0	3.44
		γ	859	0.028	0.18		
323	3.3×10^{-4}	α'	1668	0.022	0.17	111.9	6.20
		γ	1549	0.064	0.56		
296	3.3×10^{-6}	α'	2053	0.036	0.25	122.0	4.54
		γ	1412	0.049	0.50		
296	10^{-2}	α'	1924	0.033	0.38	94.5	3.95
		γ	1300	0.046	0.42		

Fig. A.3 shows the estimation results of (a,c,e) the SS curves of individual γ and α' phases and (b,d,f) the change in the volume fraction of the deformation-induced α' phase ($f(\alpha')$) during tensile testing at a strain rate of $3.3 \times 10^{-4} s^{-1}$ and temperatures of (a,b) 243 K, (c,d) 296 K, and (e,f) 323 K. The calculated overall SS curve of TRIP steel indicated by the black solid line in Fig. A.3(a,c,e) closely reproduces the experimental data (open circles) [3]. The results in Fig. A.3 suggest that as the temperature increases, the flow stress of the α' phase decreases, whereas the flow stress of the γ phase increases. Meanwhile, Fig. A.4 shows the estimation results of (a,c,e) the SS curves of individual γ and α' phases and (b,d,f) the change in $f(\alpha')$ at a temperature of 296 K and strain rates of (a,b) $3.3 \times 10^{-6} s^{-1}$, (c,d) $3.3 \times 10^{-4} s^{-1}$, and (e,f) $10^{-2} s^{-1}$. The calculated overall SS curve of TRIP steel indicated by the black solid line in Fig. A.4(a,c,e) closely reproduces the experimental data (open circles) [3]. The results in Fig. A.4 suggest that as the strain rate increases, the flow stress of the α' phase decreases, whereas the flow stress of the γ phase remains almost unchanged. The dashed line shown in Figs. A.3(b,d,f) and A.4(b,d,f) represents the estimated change in $f(\alpha')$, generally matching the experimental data (open circles) from the literature [3]. The estimated values of $f(\alpha')$ deviate systematically from the experimental values, which may be attributed to the fact that the phase fraction analysis using X-ray or neutron diffraction in multiphase TRIP steels depends on the orientation of the measured sample [21,40].

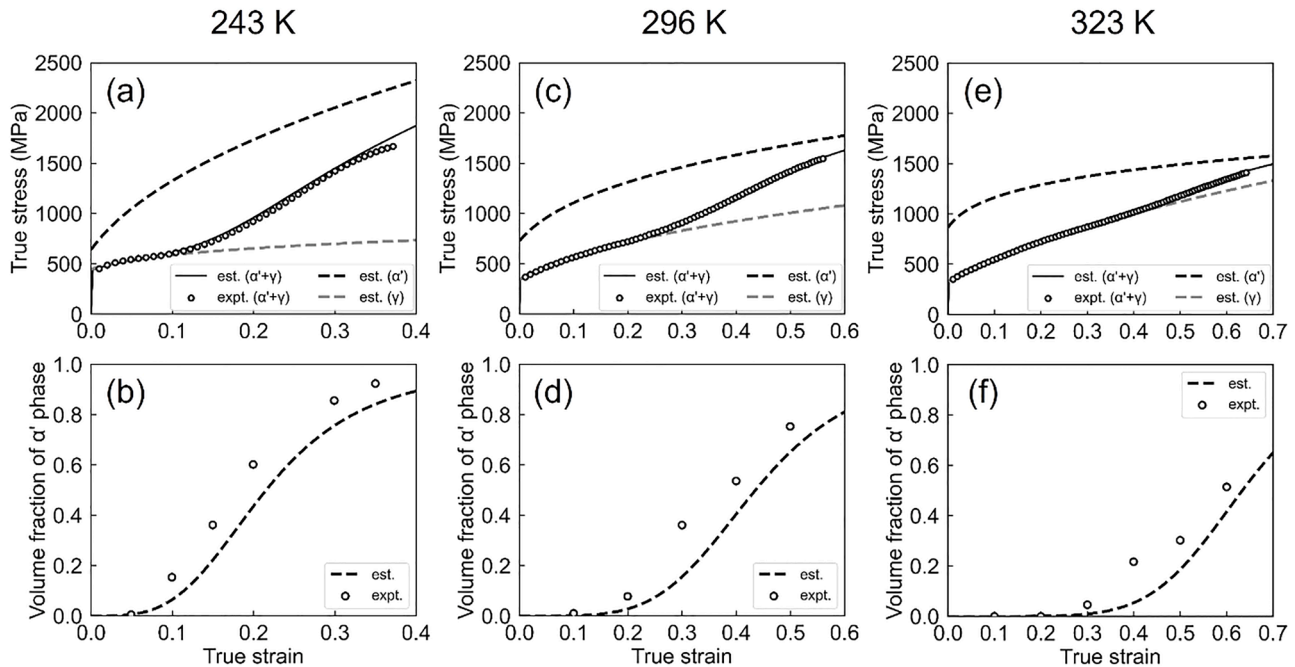


Fig. A.3. Estimation results for (a,c,e) the flow stresses of individual γ and α' phases and (b,d,f) the change in the volume fraction of the deformation-induced α' phase during tensile testing of SUS301L steel at a strain rate of $3.3 \times 10^{-4} s^{-1}$ and temperatures of (a,b) 243 K, (c,d) 296 K, and (e,f) 323 K. The solid lines in (a,c,e) represent the calculated overall SS curves of SUS301L steel, the dashed lines in (a–f) represent the estimation results, and the open symbols in (a–f) represent experimental data from the literature [3].

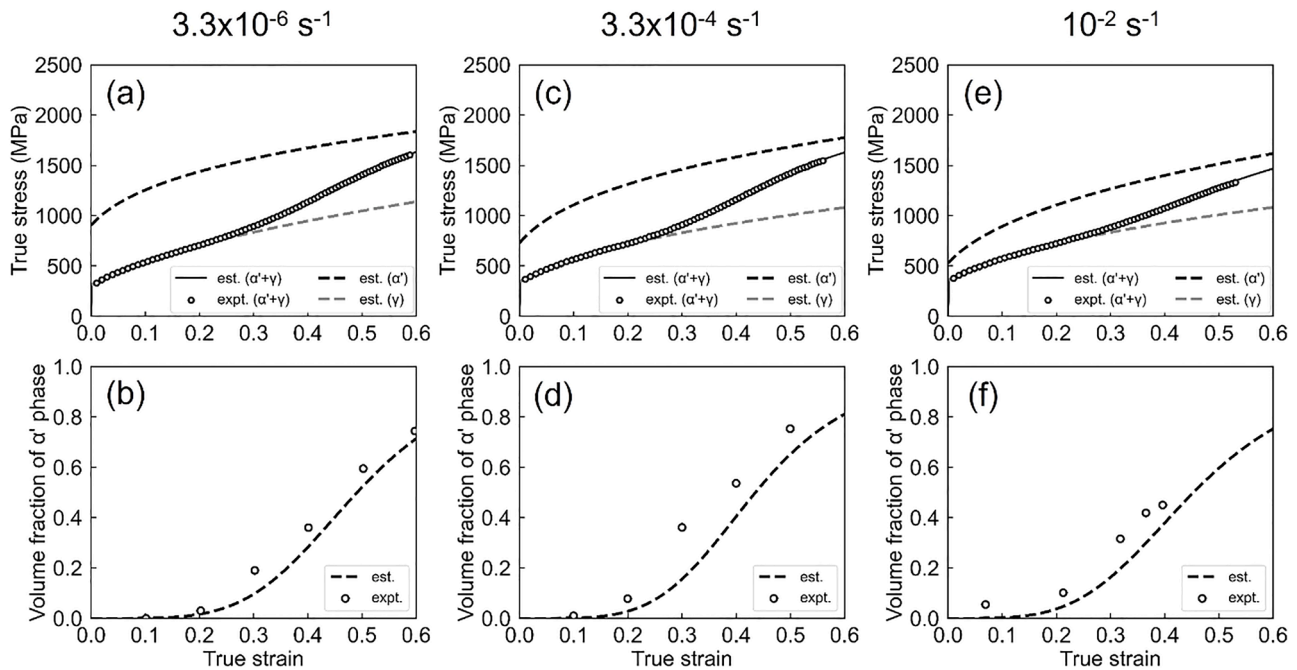


Fig. A.4. Estimation results for (a,c,e) the flow stresses of individual γ and α' phases and (b,d,f) the change in the volume fraction of the deformation-induced α' phase during tensile testing of SUS301L steel at a temperature of 296 K and strain rates of (a,b) $3.3 \times 10^{-6} \text{ s}^{-1}$, (c,d) $3.3 \times 10^{-4} \text{ s}^{-1}$, and (e,f) 10^{-2} s^{-1} . The solid lines in (a,c,e) represent the calculated overall SS curves of SUS301L steel, the dashed lines in (a–f) represent the estimation results, and the open symbols in (a–f) represent experimental data from the literature [3].

Data availability

Data will be made available on request.

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